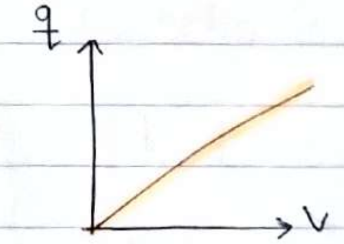
25 - Capacitance

* Electric Capacitance (C) :-

$$C = \frac{q_{\text{on a Conducter}}}{V_{\text{of the Conducter}}} \quad C/V \text{ "Farad"}$$

* Electric Capacitan : is the amount of charge needed to rase V of the conducter By 1 volt.

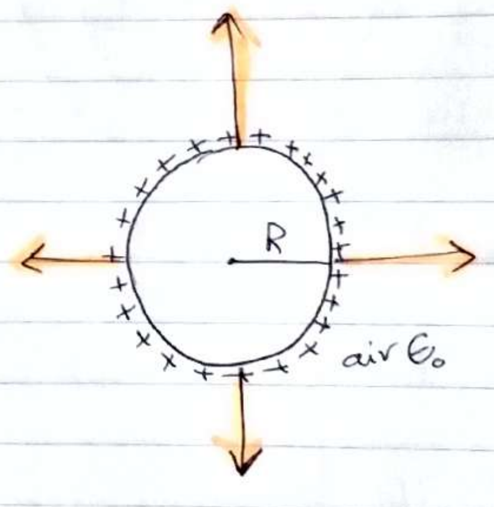


$q = CV$

$C = \text{slope}$

* conducting Sphere :-

find C ?



$V = \frac{q}{4\pi\epsilon_0 R}$ .... (1)

$C = \frac{q}{V}$ .... (2) (1) in (2)

$C = 4\pi\epsilon_0 R$

↳ depend on Geomatry

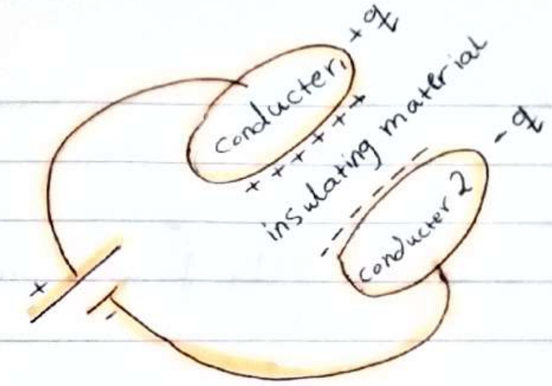
- let $R = 10\,cm$, find C ?

$C = 4\pi\epsilon_0 (10 \times 10^{-2})$

$C = 11.1\,pF$

$P = 10^{-12}$

Capaciter :-



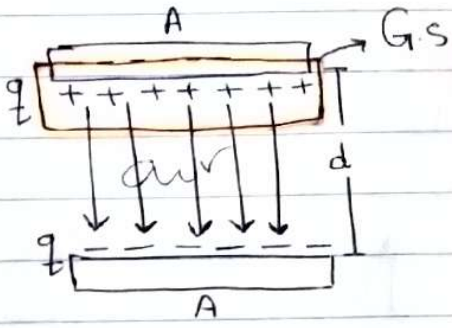
• To store :-

q ⟵ ⟶ U

1 Parallel plats Capacitor :-

• it Consist from 2 Conducting parallel plates each of Area = A.

• d is the distance between them is filled air (ϵ_0).



⇒ Find C ?

To find C, we need to find E first so

$$\int \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$E A \cos 0 = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{\epsilon_0 A}$$

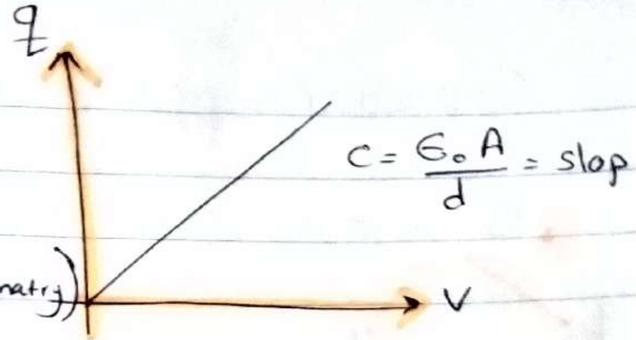
⇒ Find V ?

$$V_{\ominus} - V_{\oplus} = - \int_{+}^{-} \vec{E} \cdot d\vec{r}$$ ⤴ $\cos 0 = +$

$$V_{\oplus} - V_{\ominus} = - E dr$$

$$\Rightarrow V = Ed$$

$\Rightarrow C = \frac{q}{V} = \frac{q}{Ed}$

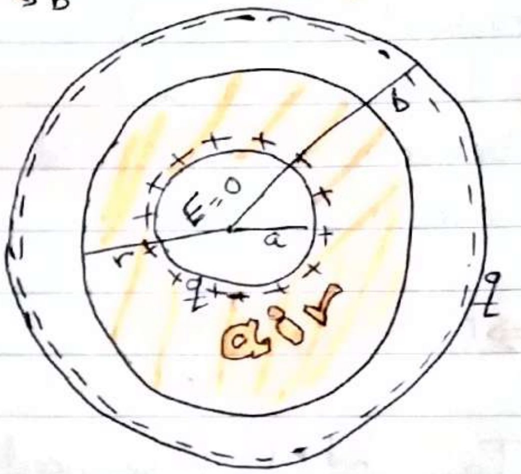


$C = \frac{\epsilon_0 A}{d}$ (c depend on Geomatry)

2 Spherical capacitor :-

- 2 conducting concentric sphere $<^{a}_{b}$ find C ?

① To find E :-



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$E \, 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

② Find ΔV :

$$V_{\ominus} - V_{\oplus} = -\int_a^b \vec{E} \cdot d\vec{r}$$

$$V_{\ominus} - V_{\oplus} = -\int_a^b \frac{q \, dr \cos 0}{4\pi\epsilon_0 r^2}$$

$$V_{\ominus} - V_{\oplus} = +\frac{q}{4\pi\epsilon_0} \left[\frac{1}{r}\right]_a^b$$

$$V_{\ominus} - V_{\oplus} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{b} - \frac{1}{a}\right)$$

$$V_{\ominus} - V_{\oplus} = \frac{q}{4\pi\epsilon_0} \frac{(a-b)}{ab}$$

$$V_{\oplus} - V_{\ominus} = \frac{q}{4\pi\epsilon_0} \left(\frac{b-a}{ab}\right)$$

$$V = \frac{q}{4\pi\epsilon_0} \frac{(b-a)}{ab}$$

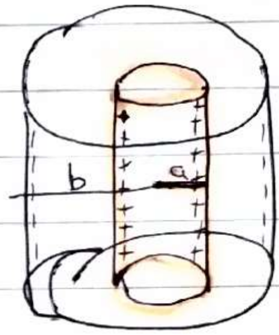
③ $$C = \frac{q}{V}$$

$$C = \frac{4\pi\epsilon_0 ab}{b-a}$$ depend on Geomatry.

3 Cylindrical Capacitor :.

- 2 Concentric conducting cylinders (a, b), Find C?

① $$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{q}{2\pi\epsilon_0 \ell r}$$



② $$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

$$\Delta V = \frac{q}{2\pi\epsilon_0 \ell} \int_a^b \frac{dr}{r}$$

$$\Delta V = \frac{q}{2\pi\epsilon_0 \ell} \ln\left(\frac{b}{a}\right)$$

③ $$C = \frac{q}{V} = \frac{2\pi\epsilon_0 \ell}{\ln\left(\frac{b}{a}\right)}$$, depend on Geomatry.

→ Electric potential Energy stored in a capacitor

= W done in charging C

= Area under the Curve q vs V

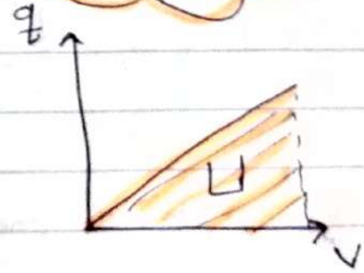
$= \frac{1}{2} q V$

→ $U = \frac{1}{2} q V$ J

$U = \frac{1}{2} \frac{q^2}{C}$ J

$U = \frac{1}{2} C V^2$ J

$C = \frac{q}{V}$



→ Energy density :-

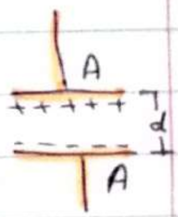
• Electric Energy density = $\frac{\text{energy}}{\text{Volume}}$

$$u = \frac{U}{V} = \frac{1/2\, C V^2}{A d}$$

$$= \frac{1/2 \left(\frac{\epsilon_0 A}{d}\right) (V^2)}{A d}$$

$$= 1/2\, \epsilon_0 \left(\frac{V}{d}\right)^2$$

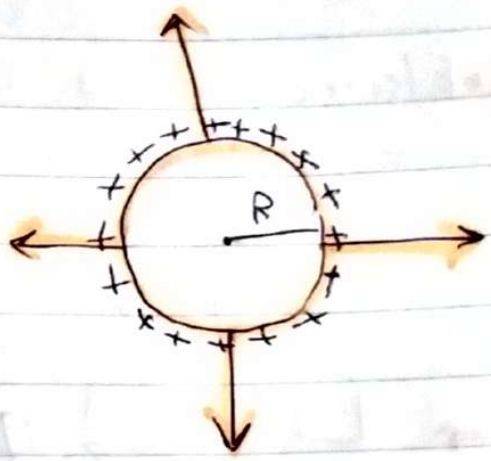
$$= 1/2\, \epsilon_0\, E^2$$



• $V = E d$

• $C = \frac{\epsilon_0 A}{d}$

Example :- 25.04



conducting Charged sphere

$R = 6.85$ cm

$q = 1.25$ nC

(a) Find U ?

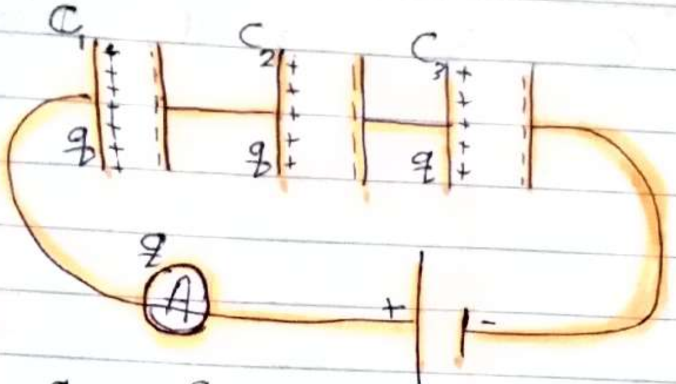
$$U = \frac{1}{2}\frac{q^2}{C} = \frac{q^2}{2(4\pi\epsilon_0 R)} = 103 \text{ nJ}$$

(b) Find the energy density at the surface of the sphere?

$$u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left[\frac{q}{4\pi\epsilon_0 R^2}\right]^2$$

$$= 25.4 \ \mu J/m^3$$

Capacitors In series:



$$q = q_1 = q_2 = q_3$$

$$C = \frac{q}{V}$$

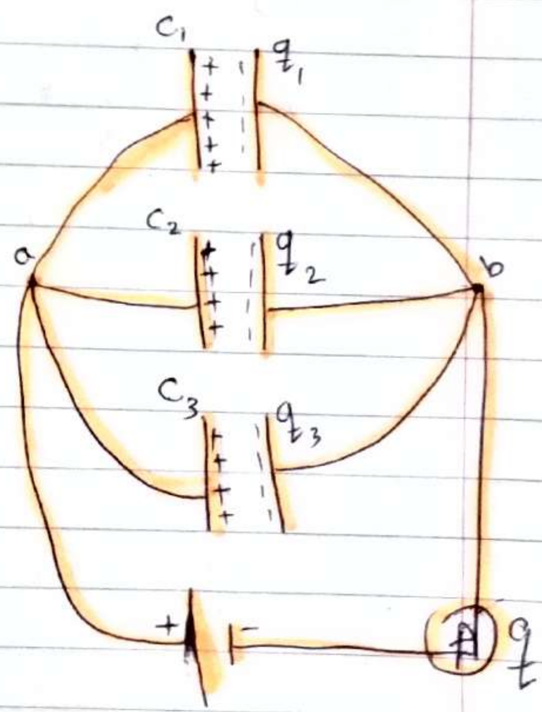
$$V = V_1 + V_2 + V_3$$

$$V = \frac{q_1}{C_1} + \frac{q_2}{C_2} + \frac{q_3}{C_3}$$

$$V = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{V}{q} = \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitors In parallel:



$$V_1 = V_2 = V_3 = V$$

$$q = q_1 + q_2 + q_3$$

$$q = C_1 V + C_2 V + C_3 V$$

$$\frac{q}{V} = C_1 + C_2 + C_3$$

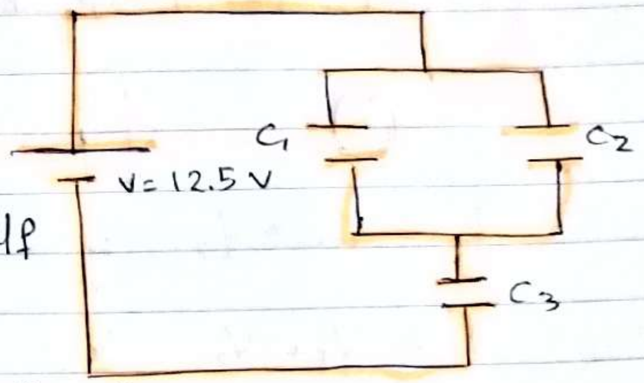
$$C_{eq} = C_1 + C_2 + C_3$$

$$C_{eq} = \sum C_i$$

• Sample problem 25.02

$C_1 = 12\ \mu f$ / $C_2 = 5.3$ / $C_3 = 4.5\ \mu f$

(a) Find C_{eq}?



$C_{12} = 12 + 5.3 = 17.3\ \mu f$

$C_{12} \parallel C_3 = \frac{17.3 \times 4.5}{(17.3 + 4.5)} = 3.57\ \mu f$

(b) Find q_1? q_2? q_3? V_1? V_2? V_3?

$q_{123} = C_{123}\ V$

$= 3.57 \times 12.5 = 44.6\ \mu c$

$q_{123} = q_{12} = q_3 = 44.6\ \mu c$

$q_3 = 44.6\ \mu c$

$V_3 = \frac{q_3}{C_3} = \frac{44.6}{4.5} = 9.9\ V$

$V_{12} = 12.5 - 9.9 = 2.6\ V$

$V_{12} = V_1 = V_2 = 2.6\ V$

$q_1 = V_1 C_1 = 31.2\ \mu c$

$q_2 = C_2 V_2 = 13.7\ \mu c$

- Capacitors with Dielectric material:-

dielectric material
- → $\epsilon > \epsilon_0$
- → dielectric constant $K = \frac{\epsilon}{\epsilon_0} > 1$
- → dielectric strength $= E_{max}$ $\frac{MV}{m}$ (→ mega)

- Increasing dielectric material in any capacitor will increas C:-

$$C_0 = \frac{\epsilon_0 A}{d} \rightarrow C = K \epsilon_0 A$$

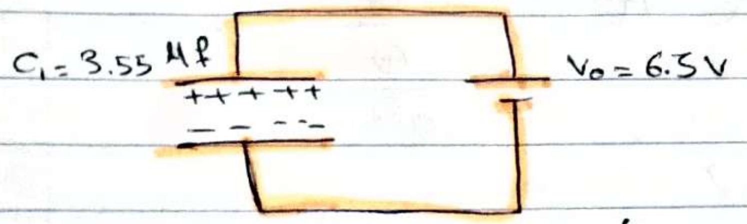
$$U = \frac{1}{2} \epsilon_0 E^2 \rightarrow u = \frac{1}{2} K \epsilon_0 E^2$$

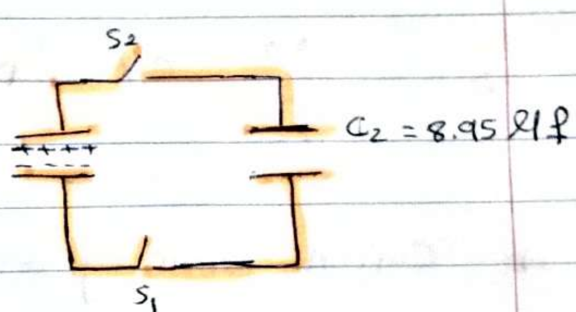
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \rightarrow \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{K\epsilon_0}$$

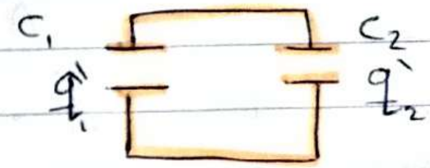
$$\epsilon_0 \rightarrow K\epsilon_0$$

• Sample problem 25.03:-

One capacitor Charging up capacitor.







$q_0 = C_1 V_0$ / $q = C_1 V_0$

$q_0 = 22.37\ \mu C$

$q_{before} = q_{after}$

$22.4 = q'_1 + q'_2$ —— (1)

$\frac{q'_1}{C_1} = \frac{q'_2}{C_2} \Rightarrow \frac{q'_1}{3.55} = \frac{q'_2}{8.95}$ —— (2)

$q'_1 = 6.35\ \mu c$

$q_2 = 16.0\ \mu c$